Study Guide

Absolute Value Solve
02/29/2012

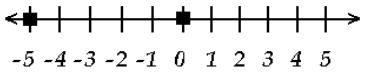
Absolute Value: Solve

The absolute value of a real number is the distance the real number x is from 0 on a number line. The absolute value of a real number is denoted by placing the real number within two vertical lines: |x|. In other words, |-5| ("the absolute value of -5") is 5 because -5 is 5 units from 0 on a number line.

There are two major principles of absolute value:

1. The absolute value of a negative number is always a positive number.
2. The absolute value of 0 is 0.

One way to find the absolute value of single digit numbers such as |-5|, is to draw a number line and plot -5 and 0 on the number line.



Notice that -5 is 5 points from 0, so the absolute value of |-5| is 5.

The student must also be able to find the absolute value of single digit numbers, solve absolute value equations, evaluate absolute value expressions, and recognize absolute value expressions on number lines.

The following example demonstrates how to solve equations with absolute value.

Example 1: Solve: |2x - 3| = 17

$$\underset{(1)}{2x-3=17 \text{ and } 2x-3=-17}$$

$$\begin{array}{ll} \underset{(2)}{2x-3=17} & \underset{(3)}{2x-3=-17} \\ \quad +3 \quad +3 & \quad +3 \quad +3 \\ \hline \frac{2x}{2}=\frac{20}{2} & \frac{2x}{2}=\frac{-14}{2} \\ x=10 & x=-7 \end{array}$$

Step 1: Set the equation up to equal both 17 and -17. This is because 2x - 3 can equal either 17 or -17 to make the original equation true.
Step 2: Solve the equation 2x - 3 = 17. Add 3 to both sides of the equation to get 2x = 20. Then divide each side of the equation by 2 to get x = 10.
Step 3: Solve the equation 2x - 3 = -17. Add 3 to both sides of the equation to get 2x = -14. Then divide each side of the equation by 2 to get x = -7.

The solutions for |2x - 3| = 17 are **x = 10 or x = -7**. However, to be sure, always check solutions by substituting both solutions (10 and -7) for x (in |2x - 3| = 17). For x = 10 (shown in Step 4) and for x = -7 (shown in Step 5):

$$\begin{array}{ll} \underset{(4)}{2(10)-3=17} & \underset{(5)}{2(-7)-3=-17} \\ 20-3=17 & -14-3=-17 \\ 17=17 & -17=-17 \end{array}$$

Example 2: Evaluate expression |x - 3| when x = -2.

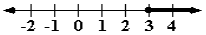
(1) |-2 - 3|
(2) |-5| = 5

Step 1: Substitute -2 in the absolute value expression for x.
Step 2: The expression |-2 - 3| becomes |-5|. The absolute value of |-5| is 5 because -5 is 5 points from 0 on a number line.

It may be useful to review the inequality symbols.

Symbol	Definition	Type of dot on number line
$<$	is less than	open
$>$	is greater than	open
$=$	is equal to	closed
$\leq$	is less than or equal to	closed
$\geq$	is greater than or equal to	closed
$\neq$	is not equal to	open

Inequalities can be represented as a value on a number line. The following number line represents the inequality $x \geq 3$.



Example 3: Solve and graph the inequality. |3x + 5| < 10

$$3x+5<10 \quad \text{and} \quad 3x+5>-10 \qquad (1)$$

(2)
$$3x+5<10$$
$$-5 \quad -5$$
$$\frac{3x}{3}<\frac{5}{3}$$
$$x<\frac{5}{3}$$

(3)
$$3x+5>-10$$
$$-5 \quad -5$$
$$\frac{3x}{3}>\frac{-15}{3}$$
$$x>-5$$

Step 1: Set up the inequality as being less than 10 and greater than -10. (The second inequality should have the "is greater than" symbol because: when we make the 10 negative, we turn the inequality symbol the opposite direction. In this case, we change it from "is less than" to "is greater than.")
Step 2: Solve the inequality 3x + 5 < 10. Subtract 5 from both sides of the inequality, then divide both sides of the inequality by 3.
Step 3: Solve the inequality 3x + 5 > -10. Subtract 5 from both sides of the inequality, then divide both sides of the inequality by 3.

The answer is: x < 5/3 and x > -5, which can also be written as -5 < x < 5/3.

Now, graph the inequality. Place an open dot on the -5 and an open dot on 5/3 because the inequality is strictly less than 5/3 and strictly greater than -5 (x cannot equal 5/3 or -5). Shade the portion of the graph that is greater than -5 and less than 5/3.